EARLY TIME AIR FIREBALL MODEL FOR A NEAR-SURFACE BURST

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Science Applications, Incorporated

Prepared for:

Defense Nuclear Agency

March 1973

DNA3029T
DATE: March, 1973
SAI-72-588-LJ

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Topical Report

This work was supported by
The Defense Nuclear Agency under:
NWED Subtask SA001-36.

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for

DEFENSE NUCLEAR AGENCY

Contract No. DNA001-73-C-0012.

UNCLASSIFIED

Security Classification

DOCUMENT CONTROL DATA - R & D

(Security classification of title, body of abstract and indexing annotation must be entered when the overall report is classified)

1. ORIGINATING ACTIVITY *(Corporate author)*

SCIENCE APPLICATIONS, INC.
1250 Prospect Street, P. O. Box 2351
La Jolla, California 92037

2a. REPORT SECURITY CLASSIFICATION

UNCLASSIFIED

2b. GROUP

3. REPORT TITLE

"Early Time Air Fireball Model for a Near-Surface Burst"

4. DESCRIPTIVE NOTES *(Type of report and inclusive dates)*

Topical Report

5. AUTHOR(S) *(First name, middle initial, last name)*

Gerald C. Pomraning

6. REPORT DATE	7a. TOTAL NO. OF PAGES	7b. NO. OF REFS
March, 1973	32	None

8a. CONTRACT OR GRANT NO.	9a. ORIGINATOR'S REPORT NUMBER(S)
DNA001-73-C-0012	SAI-72-588-LJ
b. PROJECT NO. NWED:QAXS	
c. Task and Subtask: A001	9b. OTHER REPORT NO(S) *(Any other numbers that may be assigned this report)*
d. Work Unit: 36	DNA3029T

10. DISTRIBUTION STATEMENT

Approved for Public Release; Distribution Unlimited.

11. SUPPLEMENTARY NOTES	12. SPONSORING MILITARY ACTIVITY
	Director Defense Nuclear Agency Washington, D. C. 20305

13. ABSTRACT

An integral part of the 1D-2D synthesis technique used to estimate the energy coupled into the ground from a near-surface burst is a simplified air fireball model. One of the main limitations of the model currently in use is the assumption that the device output is a square wave in time; i.e., the device radiates at a constant rate for a finite period of time. Analytic modeling work in the coupling problem has indicated that the time dependence of the device output is an important parameter in determining the radiative energy coupled into the ground. This indication, together with the fact that a square wave is a poor representation of the output of a device, led us to reformulate the current air model for a more realistic device radiative yield. During the course of this reformulation, several other improvements were made to the earlier model.

The purpose of this report is to describe in some detail this reformulation of the air fireball model. As in the current model, this reformulation envisions the air "burning out" at a fixed temperature. That is, for air temperatures less than the burnout temperature, the air opacity is assumed very large, and for air temperatures greater thanthe burnout temperature the air opacity is assumed very small.

DD FORM 1 NOV 65 1473 REPLACES DD FORM 1473, 1 JAN 64, WHICH IS OBSOLETE FOR ARMY USE.

UNCLASSIFIED

Security Classification

14. KEY WORDS	LINK A		LINK B		LINK C	
	ROLE	WT	ROLE	WT	ROLE	WT
GROUND COUPLING						
SURFACE BURST						
AIR BURNOUT						
AIR FIREBALL						

DNA3029T
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CONTENTS

1. INTRODUCTION

An integral part of the 1D-2D synthesis technique used to estimate the energy coupled into the ground from a near-surface burst is a simplified air fireball model. The current model used in ground coupling work was developed by Pyatt, et al. in 1969. One of the main limitations of this model is the assumption that the device output is a square wave in time; i.e., the device radiates at a constant rate for a finite period of time. Analytic modeling work in the coupling problem has indicated that the time dependence of the device output is an important parameter in determining the radiative energy coupled into the ground. This indication, together with the fact that a square wave is a poor representation of the output of a device, led us to reformulate the Pyatt air model for a more realistic device radiative yield. During the course of this reformulation, several other improvements were made to the earlier model.

The purpose of this report is to describe in some detail our reformulation of the air fireball model. We shall assume the reader is familiar with the Pyatt model in general terms. As in this early model, our reformulation envisions the air "burning out" at a fixed temperature. That is, for air temperatures less than the burnout temperature, the air opacity is assumed very large, and for air temperatures greater than the burnout temperature the air opacity is assumed very small.

2. REPRESENTATION OF THE DEVICE OUTPUT

We assume a point source of radiation of yield Y and a time dependence, f(t) , given by

$$f(t) = \frac{1}{N} \frac{(\alpha+\beta)e^{\alpha(t-t_o)}}{\beta+\alpha e^{(\alpha+\beta)(t-t_o)}} \quad , \quad \alpha > \beta \quad . \tag{1}$$

Here N is a normalization constant such that

$$\int_0^\infty dtf(t) = 1 \quad . \tag{2}$$

This function has the properties:

a) Rises like $e^{\alpha t}$ for small t ;

b) Falls like $e^{-\beta t}$ for large t ;

c) Has a single maximum at $t = t_o$;

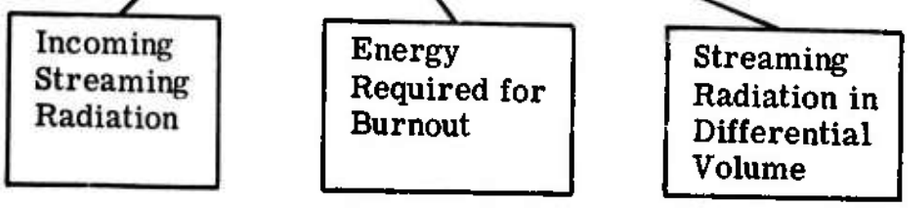
d) $f(t_o) = 1/N$.

3. BURNOUT IN INFINITE AIR

We assume that during the burnout phase of the fireball growth the air heats to a temperature T_B (the burnout temperature) and gets no hotter. We designate by Q the energy density required to heat the air to this temperature T_B. At this temperature, we assume the air has a small residual opacity $\varkappa$. The fact that $\varkappa \neq 0$ means that a thermal radiation field will build up. Let $R = R(t)$ by the fireball radius as a function of time. We perform an energy balance on an element dR, assuming the streaming (source) radiation is responsible for the air burnout at the edge of the fireball.

Energy Balance in dR in time dt

$$4\pi R^2 c E_s dt = 4\pi R^2 dR Q + 4\pi R^2 dR E_s \quad . \tag{3}$$



Here c is the speed of light, E_s is the energy density at the edge of the fireball, and all other symbols are as previously defined. Solving Eq. (3) for dR/dt, we find

$$\frac{dR}{dt} = \frac{U}{(U/c) + 4\pi R^2 Q} \quad , \tag{4}$$

where we have defined

$$U \equiv 4\pi R^2 E_s c \quad . \tag{5}$$

The boundary condition on Eq. (4) is that the fireball has a zero initial radius, i.e.,

$$R(0) = 0 \quad . \tag{6}$$

We now consider the buildup of thermal radiation in the burnout sphere. If we assume the sphere to be optically thin, then we can ignore spatial gradients and write the rate equation for the volume as a whole.

Energy Balance for Thermal Radiation

$$\frac{d}{dt}\left(\frac{4}{3}\pi R^3 E_T\right) = \rho\varkappa c\frac{4}{3}\pi R^3\left(aT_B^4 - E_T\right) \quad . \tag{7}$$

Rate of Change of Thermal Radiation	Emission of Thermal Radiation	Absorption of Thermal Radiation

The solution of Eq. (7) is, making use of Eq.(16),

$$\frac{4}{3}\pi R^3 E_T = \rho\varkappa caT_B^4 \int_0^\infty dt' \frac{4}{3}\pi R^3(t') e^{-\rho\varkappa c(t-t')} \quad . \tag{8}$$

Here the new symbols introduced are E_T , the thermal radiation density, ρ , the air density, and a , the radiation constant.

Finally, we consider the streaming radiation. Since at the burnout temperature T_B the air opacity $(\varkappa)$ is non-zero, the burned out air will radiate and attempt to cool. We assume the streaming radiation will be absorbed by the air as it radiates to maintain its temperature. We perform an energy balance at some radius r in the interior of the fireball.

Energy Balance for E_s In dr in time dt

$$4\pi r^2 E_s c\,dt\big|_r - 4\pi r^2 E_s c\,dt\big|_{r+dr} = -4\pi r^2 dr\,dE_s \quad . \tag{9}$$

Streaming Energy In	Streaming Energy Out	Streaming Energy Absorbed

Equation (9) gives the result

$$\frac{d(4\pi r^2 E_s c)}{dr} = 4\pi r^2 \frac{dE_s}{dt} \quad . \tag{10}$$

Now, to maintain the air at the burnout temperature, the rate of absorption of streaming energy must equal the net rate of emission of thermal energy. Thus we have

$$-\frac{dE_s}{dt} = \rho \varkappa c(aT_B^4 - E_T) \quad , \tag{11}$$

and Eq. (10) becomes

$$\frac{d(4\pi r^2 E_s c)}{dr} = -4\pi r^2 \rho \varkappa c(aT_B^4 - E_T) \quad . \tag{12}$$

The boundary condition on Eq. (12) is

$$4\pi r^2 E_s(r,t)c \xrightarrow[r\to 0]{} Yf(t) \quad . \tag{13}$$

Equation (13) is just the energy conservation condition for an arbitrarily small sphere surrounding the source.

We rewrite Eq. (12) as

$$\frac{d}{dr}[4\pi r^2 E_s(r,t)c] = -4\pi r^2 S(t) \quad , \tag{14}$$

where we have defined

$$S(t) \equiv \rho \varkappa c[aT_B^4 - E_T(t)] \quad . \tag{15}$$

Now, r in Eq. (14) is a path length variable and t is the associated time at which the radiation under consideration is at the path length position r. Hence in Eq. (14)

$$t = t_{ref} + \frac{r}{c} \quad , \tag{16}$$

where t_{ref} is any fixed reference time. Let us set

$$t_{ref} = t_{act} - \frac{R(t_{act})}{c} \quad , \tag{17}$$

where t_{act} is the actual time of interest. Thus Eq. (14) reads

$$\frac{d}{dr}\left[4\pi r^2 E_s(r, t - \frac{R-r}{c})c\right] = -4\pi r^2 S(t - \frac{R-r}{c}) \quad , \tag{18}$$

where we have dropped the subscript on t_{act} . Integration of Eq. (18) from r=0 to r=R yields

$$4\pi R^2 E_s(R,t)c = 4\pi r^2 E_s(r, t - \frac{R-r}{c})c\Big|_{r=0}$$

$$- \int_0^R dr 4\pi r^2 S(t - \frac{R-r}{c}) \quad , \tag{19}$$

or, using Eq. (13),

$$4\pi R^2 E_s(R,t)c = Yf(t - \frac{R}{c}) - \int_0^R dr 4\pi r^2 S(t - \frac{R-r}{c}) \quad . \tag{20}$$

It is $E_s(R,t)$ that is denoted by simply E_s in Eq. (3).

We summarize the three pertinent equations to be solved for the fireball radius, thermal radiation density, and streaming radiation density as a function of time. These are [see Eqs. (4), (8), and (20)]

$$\frac{dR}{dt} = \frac{U}{(U/c) + 4\pi R^2 Q} \quad , \tag{21}$$

$$E_T = \frac{\rho\kappa caT_B^4}{R^3} \int_0^t dt' R^3(t') e^{-\rho\kappa c(t-t')} \quad , \tag{22}$$

$$U = Yf(t - \frac{R}{c}) - 4\pi \int_0^R dr r^2 S(t - \frac{R-r}{c}) \quad , \tag{23}$$

where [see Eqs. (5) and (15)]

$$U \equiv 4\pi R^2 E_s c \quad , \tag{24}$$

$$S \equiv \rho \varkappa c[aT_B^4 - E_T] \quad . \tag{25}$$

In general, these equations must be solved numerically, with starting conditions

$$R(0) = 0 \quad , \tag{26}$$

$$E_T(0) = 0 \quad , \tag{27}$$

$$S(0) = \rho \varkappa c a T_B^4 \quad , \tag{28}$$

$$U(0) = Yf(0) \quad , \tag{29}$$

$$dR(0)/dt = c \quad . \tag{30}$$

The numerical solution continues until

$$U = 4\pi R^2 E_s c = 0 \quad , \tag{31}$$

or

$$\frac{\frac{4}{3}\pi R^3 (Q + E_T)}{Y \int_0^t dt' f(t')} = 1 \quad . \tag{32}$$

The physical meaning of Eq. (31) is that the streaming radiation is entirely absorbed before it reaches the edge of the fireball. The physical meaning of Eq. (32) is that the energy emitted by the source is just sufficient to maintain the air at a temperature T_B together with the thermal radiation field which has built up. We shall discuss the finite difference analogue of Eqs. (21) through (23) in a later section in this report.

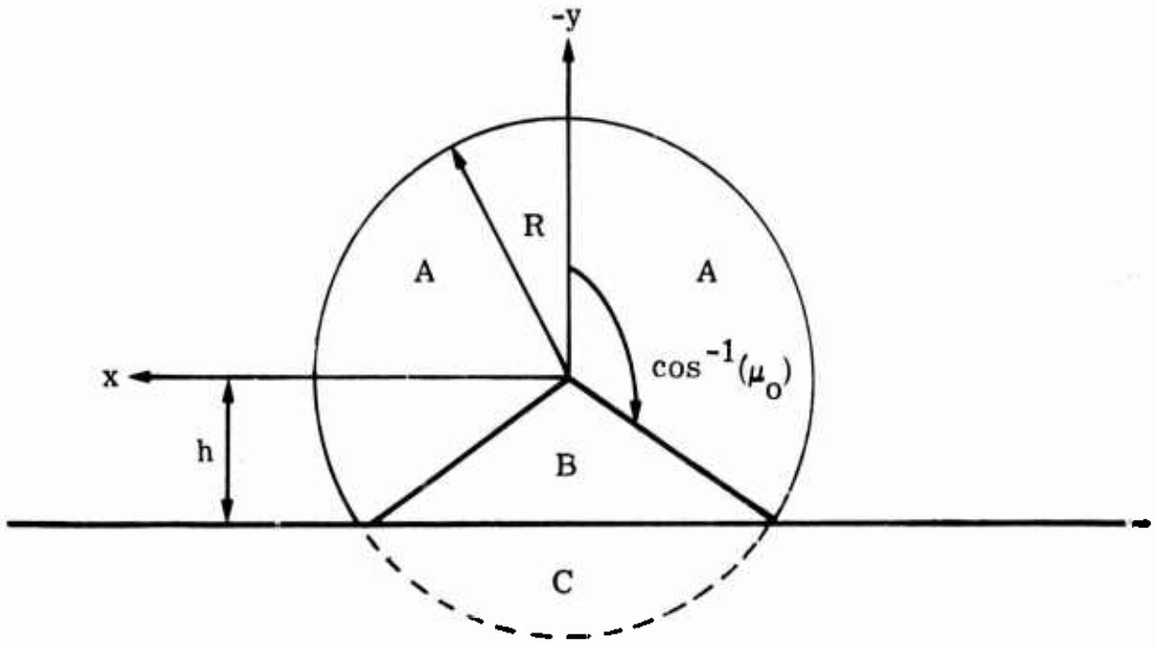
A quantity we shall find useful later on is the effective temperature of the thermal radiation field, defined by the relationship

$$aT_{eff}^4 = E_T \quad . \tag{33}$$

4. BURNOUT IN THE PRESENCE OF A GROUND

We now modify the results of the last section to include the influence of the ground. The fireball will remain spherical until its radius equals h, the height of burst. At later times, we assume the fireball is a truncated sphere, truncated by the air-ground interface.

At such a time when $R>h$, we have the schematic



Let us compute the volume of various regions of this sphere. We have for all three regions the obvious result

$$V_{ABC} = \frac{4}{3}\pi R^3 \quad . \tag{34}$$

The volume of regions A and B is given by

$$V_{AB} = \int_{-R}^{h} dy \pi x^2 = \int_{-R}^{h} dy \pi (R^2 - y^2) \quad , \tag{35}$$

$$V_{AB} = \pi\left(\frac{2}{3}R^3 + R^2h - \frac{h^3}{3}\right) \quad . \tag{36}$$

The volume of region A alone is given by

$$V_A = 2\pi \int_0^R dr r^2 \int_{\mu_o}^1 d\mu = \frac{2\pi R^3}{3} \int_{-h/R}^1 d\mu \quad , \tag{37}$$

$$V_A = \frac{2}{3}\pi R^2(R + h) \quad . \tag{38}$$

Now, we assume that the yield directed into segment B is not absorbed in the air, but is entirely absorbed in the ground and then entirely reradiated back into the fireball, all in zero time. This energy is then available to increase the size of the fireball in the air. We account for this by increasing the yield. We let

$$Yf(t) \longrightarrow Yf(t)d(t) \quad , \tag{39}$$

where d(t) is the enhancement factor given by the ratio of 4π to the solid angle associated with Region A. Thus we have

$$d(t) = \frac{V_{ABC}}{V_A} = \frac{2R}{R+h} \quad . \tag{40}$$

Of course, Eq. (40) holds only for $R \geq h$. For $R \leq h$, we have $d(t) \equiv 1$.

The burnout equations in the presence of a ground then become [see Eqs. (21) through (32)]

$$\frac{dR}{dt} = \frac{U}{(U/c) + 4\pi R^2 Q} \quad , \tag{41}$$

$$E_T = \frac{\rho\varkappa caT_B^4}{V_{AB}} \int_0^t dt' V_{AB}(t') e^{-\rho\varkappa c(t-t')} \quad , \tag{42}$$

$$U = Yd(t - \frac{R}{c})f(t - \frac{R}{c}) - 4\pi \int_0^R drr^2 S(t - \frac{R-r}{c}) \quad , \tag{43}$$

with U and S still defined by Eqs. (24) and (25) and

$$V_{AB} = \begin{cases} \frac{4}{3}\pi R^3 \quad , & R \leq h \\ \pi(\frac{2}{3}R^3 + R^2 h - \frac{h^3}{3}) \quad , & R \geq h \quad . \end{cases} \tag{44}$$

In writing Eq. (42) we have replaced $\frac{4}{3}\pi R^3$ in Eq. (22) with V_{AB}, the actual fireball volume. The burnout phase of the fireball growth is over when

$$U \equiv 4\pi R^2 E_s c = 0 \quad , \tag{45}$$

or

$$\frac{V_{AB}(Q + E_T)}{Y\int_0^t dt' f(t')} = 1 \quad . \tag{46}$$

The significance of Eqs. (45) and (46) has been discussed previously.

5. DIFFUSION PHASE OF THE FIREBALL GROWTH

Following the end of burnout, we assume that further fireball growth takes place by a diffusion process. At the end of burnout, the material and thermal radiation field are in general not in equilibrium, and we formulate the diffusion process to take this into account. We assume that the diffusion growth of the fireball is described by one dimensional spherical equation, but we shall account for the intersection of the fireball with the ground.

In spherical geometry, the energy conservation equation is

$$\frac{\delta}{\delta t}(c_v T + E_T + E_s) + \frac{1}{r^2}\frac{\delta}{\delta r}(r^2 F) = \frac{Yf(t)}{4\pi r^2}\delta(r) \quad , \tag{47}$$

where c_v is the air heat capacity (assumed constant), F is the radiative flux, and $\delta(r)$ is the Dirac delta function indicating the source of radiation is at the center of the sphere. To obtain diffusion theory, we assume the radiative flux is proportional to the gradient of the thermal radiation energy density; in particular we write

$$F = -\frac{c\lambda(T)}{3}\frac{\delta E_T}{\delta r} \tag{48}$$

where $\lambda(T)$ is the air mean free path which, as indicated, depends upon temperature. We further assume that $\lambda(T)$ has a cubic temperature dependence, i.e.,

$$\lambda(T) = \lambda\left(\frac{T}{T_B}\right)^3 \tag{49}$$

where λ, a constant, is the air mean free path at T_B, the burnout temperature. Use of Eqs. (48) and (49) in Eq. (47) yields

$$\frac{\partial}{\partial t}(c_v T+E_T+E_s) = \frac{1}{r^2}\frac{\partial}{\partial r}\left[r^2\frac{c\lambda}{3}\left(\frac{T}{T_B}\right)^3\frac{\partial E_T}{\partial r}\right] + \frac{Yf(t)\delta(r)}{4\pi r^2} \quad . \tag{50}$$

Finally, we assume

$$E_T = \epsilon a T^4 \quad , \tag{51}$$

where ϵ is allowed to depend upon time but not space. [If ϵ were also allowed to depend upon space, Eq. (51) could be considered a definition for ϵ .] Eq. (51) allows us to write

$$T^3\frac{\partial E_T}{\partial r} = \frac{4}{7}\frac{\partial}{\partial r}(a\epsilon T^7) = \frac{4}{7}\frac{\partial}{\partial r}(T^3 E_T) \quad , \tag{52}$$

and use of Eq. (52) in Eq. (50) gives the diffusion equation

$$\frac{\partial}{\partial t}(c_v T+E_T+E_s) = \frac{4c\lambda}{21T_B^3}\frac{1}{r^2}\frac{\partial}{\partial r}\left[r^2\frac{\partial}{\partial r}(T^3 E_T)\right] + Y\frac{f(t)\delta(r)}{4\pi r^2} \quad . \tag{53}$$

To proceed further, we form the first two spatial moments of Eq. (53). Integration over all volume yields

$$\frac{\partial}{\partial t}\int_0^\infty dr\, 4\pi r^2(c_v T+E_T+E_s) = Yf(t) \quad . \tag{54}$$

We now assume T, E_T, and E_s to be space independent within the fireball and zero elsewhere. This gives

$$\frac{\partial}{\partial t}\left[\frac{4}{3}\pi R^3(c_v T+E_T+E_s)\right] = Yf(t) \quad . \tag{55}$$

We recognize $\frac{4}{3}\pi R^3$ as the volume of a sphere, and interpret this volume as the fireball volume, V_{AB} [see Eq. (44)].

Thus Eq. (55) becomes

$$\frac{\partial}{\partial t}\left[V_{AB}(c_v T+E_T+E_s)\right] = Yf(t) \quad . \tag{56}$$

Equation (56) is, of course, just the conservation equation. Similarly, multiplying Eq. (53) by r^3 and integrating over all space, we obtain

$$\frac{\partial}{\partial t}\int_0^\infty dr r^3(c_v T+E_T+E_s) = -\frac{8c\lambda}{21T_B^3}\int_0^\infty dr r T^3 E_T \quad , \tag{57}$$

where the right hand side of Eq. (57) resulted from two integrations by parts. Again assuming T, E_T, and E_s constant in space within the fireball and zero elsewhere yields

$$\frac{1}{4}\frac{\partial}{\partial t}[R^4(c_v T+E_T+E_s)] = -\frac{4c\lambda}{21T_B^3} R^2 T^3 E_T \quad . \tag{58}$$

For a sphere we have

$$R = \left(\frac{3V}{4\pi}\right)^{1/3} \quad , \tag{59}$$

and if we interpret the volume in Eq. (59) as V_{AB}, the actual fireball volume, Eq. (58) becomes

$$\frac{\partial}{\partial t}\left[V_{AB}^{4/3}(c_v T+E_T+E_s)\right] = \left(\frac{4\pi}{3}\right)^{2/3}\left(\frac{16c\lambda}{21T_B^3}\right) V_{AB}^{2/3} T^3 E_T \quad . \tag{60}$$

Equations (56) and (60) are the first two spatial moments of the diffusion equation.

The interaction between the material and radiation is handled by assuming T, E_T, and E_s satisfy equilibration equations of the type

$$\frac{\partial}{\partial t}(V_{AB} c_v T) = \rho \varkappa c\left(\frac{T_B}{T}\right)^3 V_{AB}(E_T+E_s-aT^4) \quad , \tag{61}$$

$$\frac{\partial}{\partial t}(V_{AB}E_T) = \rho\kappa c\left(\frac{T_B}{T}\right)^3 V_{AB}(aT^4-E_T) \quad , \tag{62}$$

$$\frac{\partial}{\partial t}(V_{AB}E_s) = -\rho\kappa c\left(\frac{T_B}{T}\right)^3 V_{AB}E_s + Yf(t) \quad . \tag{63}$$

The sum of Eqs. (61) through (63) is just Eq. (56), the conservation equation. We consider Eqs. (60) through (63) as four equations for the four unknowns V_{AB} , T , E_T , and E_s . These equations are solved numerically, as we shall discuss in a later section of this report, with the starting conditions on the four dependent variables corresponding to the conditions at the end of burnout. As before, it will be useful to define an effective temperature for the thermal radiation field by the relationship

$$aT_{eff}^4 = E_T \quad . \tag{64}$$

6. FLUX IMPINGING UPON THE GROUND

Once the fireball behavior as a function of time has been determined, it is a simple task to compute the flux of energy impinging upon the ground at any radial distance r ($r=0$ lies directly under the device).

Define t^* as the time when the fireball first touches the ground at r. For $t < t^*$, the flux incident on the ground at position r is identically zero. The streaming radiation energy density at r for $t > t^*$ is given by the solution of the equation [see Eq. (20)]

$$4\pi R^2 E_s c = Yf(t - \frac{R}{c}) - \int_0^R d\xi 4\pi \xi^2 S(t - \frac{R-\xi}{c}) \quad , \tag{65}$$

where R is the fireball radius at $t=t^*$ and given by

$$R = \sqrt{r^2 + h^2} \quad , \tag{66}$$

and S is given by [see Eq. (15)]

$$S = \rho \varkappa c(aT_B^4 - E_T) \quad . \tag{67}$$

Then, the streaming flux per unit area perpendicular to the ground at point r is given by

$$F_s = cE_s \, h/R \quad . \tag{68}$$

In addition to this streaming flux, the fireball thermal radiation field leads to a flux in the amount

$$F_T = \sigma T_{eff}^4 \quad , \tag{69}$$

where $\sigma = ca/4$ and T_{eff} is the effective temperature of the fireball thermal radiation [see Eqs. (33) and (64)]. Thus the total flux F

impinging upon the ground as a function of time at some position r is given by

$$F = F_s + F_T = \frac{cE_s h}{R} + \frac{ca}{4} T_{eff}^4 \quad . \tag{70}$$

7. NUMERICAL ANALYSIS OF FIREBALL GROWTH EQUATIONS

We discuss briefly the finite difference equations associated with the burnout equations, (41) through (43), and the diffusion equations, (60) through (63). The finite difference equations conserve energy exactly and are unconditionally stable in the sense that accuracy alone controls the size of the time step.

Equations (41) through (43) become, with i denoting the time index,

$$R_{i+1} = R_i + \left(\frac{dR}{dt}\right)_i (t_{i+1} - t_i) \quad , \tag{71}$$

$$E_{Ti} = \frac{\rho \varkappa c a T_B^4}{V_{ABi}} \sum_{j=1}^{i} \frac{1}{2} (t_{j+1} - t_j) \left[V_{ABj} e^{-\rho \varkappa c (t_{i+1} - t_j)} + V_{AB,j+1} e^{-\rho \varkappa c (t_{i+1} - t_{j+1})} \right] \quad , \tag{72}$$

$$U_{i+1} = Yf(t_{i+1} - \frac{R_{i+1}}{c})\, d(t_{i+1} - \frac{R_{i+1}}{c}) - 4\pi \sum_{j=1}^{i} \frac{1}{2} (R_{j+1} - R_j) \left[R_j^2 S\left(t_{i+1} - \frac{R_{i+1} - R_j}{c}\right) + R_{j+1}^2 S\left(t_{i+1} - \frac{R_{i+1} - R_{j+1}}{c}\right) \right] \quad , \tag{73}$$

$$\left(\frac{dR}{dt}\right)_{i+1} = \frac{U_{i+1}}{(U_{i+1}/c) + 4\pi R_{i+1}^2 Q} \quad , \tag{74}$$

where

$$S_i = \rho\kappa c(aT_B^4 - E_{i+1}) \tag{75}$$

and

$$V_{ABi} = \begin{cases} \frac{4}{3}\pi R_i^3 , & R_i \le h \\ \pi(\frac{2}{3}R_i^3 + R_i^2 h - \frac{h^3}{3}) . & R_i \ge h \end{cases} \tag{76}$$

To finite difference Eqs. (60) through (63), we define

$$\bar{T} = V_{AB}T , \tag{77}$$

$$\bar{E}_T = V_{AB}E_T , \tag{78}$$

$$\bar{E}_s = V_{AB}E_s , \tag{79}$$

$$A = \rho\kappa c(T_B/T)^3 , \tag{80}$$

$$\gamma = (\frac{4\pi}{3})^{2/3}(\frac{16c\lambda}{21})(\frac{T}{T_B})^3 . \tag{81}$$

Defining

$$(\Delta t)_i = t_{i+1} - t_i , \tag{82}$$

we then write the conservative finite difference equations

$$\left[V_{AB}^{1/3}(c_\gamma \bar{T}+\bar{E}_T+\bar{E}_s)\right]_{i+1} = \left[V_{AB}^{1/3}(c_v \bar{T}+\bar{E}_T+\bar{E}_s)\right]_i + \left(\frac{\gamma \bar{E}_T}{V_{AB}^{1/3}}\right)_i (\Delta t)_i , \tag{83}$$

$$c_v\left[\frac{\bar{T}_{i+1} - \bar{T}_i}{(\Delta t)_i}\right] = A_i(\bar{E}_{T,i+1} + E_{s,i+1} - aT_i^3\bar{T}_{i+1}) \quad , \tag{84}$$

$$\frac{\bar{E}_{T,i+1} - \bar{E}_{T,i}}{(\Delta t)_i} = A_i\left(aT_i^3\bar{T}_{i+1} - \bar{E}_{T,i+1}\right) \quad , \tag{85}$$

$$\frac{\bar{E}_{s,i+1} - \bar{E}_{s,i}}{(\Delta t)_i} = -A_i\bar{E}_{s,i+1} + \frac{Y}{(\Delta t)_i}\int_{t_i}^{t_{i+1}} dt'f(t') \quad . \tag{86}$$

Adding these equations, we find

$$(c_v\bar{T}+\bar{E}_T+\bar{E}_s)_{i+1} = (c_v\bar{T}+\bar{E}_T+\bar{E}_s)_i + Y\int_{t_i}^{t_{i+1}} dt'f(t') \quad , \tag{87}$$

which shows that the finite difference equations conserve energy. The integral over f(t) in Eqs. (86) and (87) is performed using a two-point exponential quadrature since f(t) varies primarily exponentially [see Eq. (1)]. Thus

$$\int_{t_i}^{t_{i+1}} dt'f(t') = \frac{(\Delta t)_i[f(t_{i+1}) - f(t_i)]}{\ell n[f(t_{i+1})/f(t_i)]} \quad . \tag{88}$$

This same quadrature formula was used in computing N , the normalization integral in Eq. (1), and hence the integration of the time dependent yield is handled consistently throughout, assuring numeric energy conservation. Rearranging Eqs. (84) through (86), we have

$$\bar{E}_{s,i+1} = \frac{\bar{E}_{s,i} + Y\int_{t_i}^{t_{i+1}} dt'f(t')}{1+A_i(\Delta t)_i} \quad , \tag{89}$$

$$\bar{T}_{i+1} = \frac{\left\{\bar{T}_i + \frac{A_i}{c_v}\left[\bar{E}_{s,i+1} + \frac{\bar{E}_{Ti}}{1+A_i(\Delta t)_i}\right](\Delta t)_i\right\}[1+A_i(\Delta t)_i]}{1 + A_i\left(1 + \frac{aT_i^3}{c_v}\right)(\Delta t)_i} \quad , \tag{90}$$

$$\bar{E}_{T,i+1} = \frac{\bar{E}_{Ti} + A_i a T_i^3 \bar{T}_{i+1}(\Delta t)_i}{1 + A_i(\Delta t)_i}$$

Equations (83) and (89) through (91) are solved for the four unknowns V_{AB}, T, E_T, and E_s.

8. DIFFERENCES OF FIREBALL MODEL FROM EARLIER (PYATT) MODEL

We list here the major differences between the fireball model described in this report and the earlier one due to Pyatt, et al.

a) The new model is formulated for arbitrary time dependence of the device output. A reasonable representation of this time dependence is given by Eq. (1).

b) During burnout, the buildup of thermal radiation is calculated assuming the fireball to be optically thin [see Eq. (7)]. The earlier model used an optically thick equation.

c) The yield enhancement factor [see Eq. (40)] is more physically reasonable than that used in the Pyatt model.

d) The earlier model uses an arbitrary power law to describe the time dependence of the fireball temperature between the time the device ceases to radiate and the time burnout is over. No such arbitrary fit is used in the current model.

e) The earlier model assumed equilibrium between the matter and the thermal radiation field during the diffusion phase of the fireball growth. The present model makes no such assumption, but allows equilibration to take place simultaneously with the diffusion [see Eqs. (61) through (63)].

f) The spatial dependence of the streaming radiation is calculated in the present model, rather than assuming a $1/r^2$ dependence as in the Pyatt model [see Eq. (20)].

g) An ad hoc similarity solution is not used to solve the diffusion equation. Rather, a solution method which correctly accounts

for both the internal and radiative energy together is used [see Eq. (53)].

h) A correction for the finite size of the fireball, used in the earlier model, is not required. The thermal radiation field is calculated consistently for all times.

9. FINAL REMARKS

If the 1D-2D synthesis technique is to be used for future energy coupling calculations, it would seem desirable to use the fireball model discussed in this report. The primary reason for this is that this model incorporates a more reasonable device output time dependence, and analytic calculations indicate that this time dependence can significantly affect the energy coupled.

We intend to couple this air fireball model with a diffusion treatment of radiative transfer in the ground, and use this model to perform parameter surveys in the energy coupling area. This will give us a capability of investigating very inexpensively the sensitivity of the energy coupled to such parameters as yield, height of burst, time dependence of the device output, and ground properties. In addition, we will be able to investigate certain aspects of the accuracy of the 1D-2D synthesis technique, such as the importance of having a correct description of the air burnout, and the number of radial slices required in the ground for good accuracy.